2. Number Patterns, Sequences and Series

A. Investigating patterns

Patterns have been a topic of interest for humans for thousands of years. Ancient civilisations, such as those of the Egyptians and the Greeks, used patterns in the design of their architecture. In modern times, mathematicians and scientists use patterns, found through experiments and problems, to discover new ideas.

Conjectures are theories about patterns.
For example – Look at this pattern → 2; 4; 8; 16; 32…. What is happening?
The next term is always double the previous term. Our conjecture
(or theory) is that you have to multiply the previous term by 2 to get
the next term. To write this as a formula we can say $T_n = 2^n$.
So to get to this formula we know that:
Term 1 = 2
Term 2 = $2 \times 2 = 2^2$
Term 3 = $2 \times 2 \times 2 = 2^3$
So you can see that the term number is the same as the exponent in each term.

Activity 2.1:

Find a formula or conjecture for each of the patterns shown below, and give the next three terms of each pattern:

a) 0; 3; 8; 15;……

b) -1; 1; -3; 3; -5; 5; ……

c) $\frac{1}{64}$; $\frac{1}{32}$; $\frac{1}{16}$; …

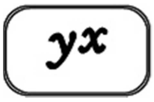
d) 1; 3; 9; 27;…

e) 0.1; 0.01; 0.101; 0.0101; ……

f) $x^2y; 2x^3y^3 ; 4x^4y^5; \ldots$

Using the calculator to solve Sequence and Series calculations:

Use the following keys from the EL-W535HT Scientific Calculator:



This key calculates exponential values

These keys together calculate the xth root of y

This key inputs a fraction or improper fraction with and numerator and denominator

These keys together input a mixed fraction

B. Quadratic Sequences

In a linear or “Arithmetic” sequence you have a first common difference – in other words you add or subtract a constant value.
For example: 4, 8, 12, 16…. and so on. You add a constant difference of 4. This is an example of a first common difference.

In a quadratic sequence you add either an increasing or decreasing amount every time. That amount always increases or decreases by a constant amount – your second common difference.
For example: 1, 3, 7, 13, 21…. and so on.

1 3 7 13 21 → sequence
2 4 6 8 → first difference
2 2 2 → second common/constant difference

The formula for a quadratic sequence is $T_n = an^2 + bn + c$: where T_n is your term value and n is your term position.

To find a, b and c you use these three formula's

a + b + c = the first term, in our example = 1

3a + b = the first "first difference", in our example = 2

2a = the second common difference, in our example = 2.

Now solve these equations from the bottom up (↑)

So: 2a = 2

a = 1

Then: 3a + b = 2

3(1) + b = 2

b = 2 – 3

b = - 1

And lastly : a + b + c = 1

1 + (-1) + c = 1

c = 1

IEB students will be familiar with this formula:

$$T_n = T_1 + (n-1)f + \frac{(n-1)(n-2)s}{2}$$

f **=** first differnence

s = second difference

$$\therefore T_n = 1 + (n-1)(2) + \frac{(n-1)(n-2)(2)}{2}$$

$$\therefore T_n = 1 + 2n - 2 + n^2 - 3n + 2$$

$$\therefore T_n = n^2 - n + 1$$

Now substitute these values into your formula:

$T_n = 1n^2 – 1n + 1$.

To check that our formula works, we choose any position for example, n = 3, and we make sure that it equals the value at that position, $T_n = 7$.

$T_3 = 1(3)^2 – 1(3) + 1 = 7$

<u>Activity 2.2</u>

1. *Find the formula for the following sequences*:

a) 2, 5, 10, 17, 26….

b) 1, 2, 7, 16, 29, 46….

c) 3, 6, 12, 21, 33….

d) 1, 3, 10, 22, 39, 61…

e) 3, 5, 8, 12, 17….

f) 2, 6, 11, 17, 24, 32…

2. *Given the sequence: 3, 4, 7, 12, 19, 28… Find:*

a) The seventh and eighth terms

b) The formula representing the sequence

c) The 22nd term

d) If T_n = 199, find n.

3. *Given the sequence: 2, 4, 9, 17…*

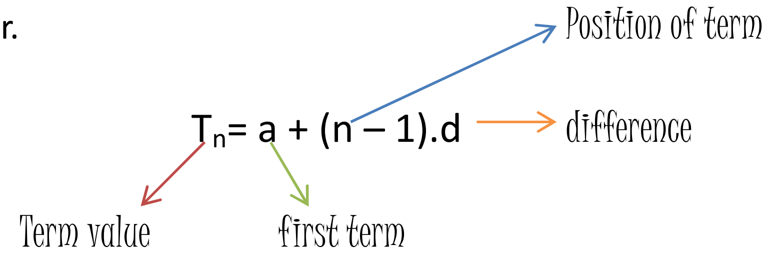
a) Continue the sequence for three more terms
b) Find the formula of the sequence
c) Find the 13^{th} term
d) If $T_n = 612$, find n.

4. A chicken farmer goes to a market in order to buy chickens. He knows that on each successive day the prices of the chickens go down. On the first day he buys 10 chickens. On the second day he buys 20 chickens. On the third day he buys 36 chickens, on the fourth day he buys 58 chickens. He continues in this pattern until the last day, when he buys 206 chickens.

a) How many days does the market go on for?
b) How many chickens does the farmer buy in total?
c) If the market continued for two weeks, how many chickens would the farmer buy on the last day?

C. Arithmetic Sequence

An ordered list is called a sequence, where T_1 is known as term 1, T_2 is known as term 2, and T_3 is known as term 3, etc. The general term is T_n, where 'n' is a natural number.



Activity 2.3

1. *Given the general term, write down the first 5 terms, as well as the 100th term of the sequence:*

a) $T_n = n - 3$ b) $T_n = 7 - n$

c) $T_n = 2n - 4$ d) $T_n = 3n + 1$

e) $T_n = -n + 2$ f) $T_n = \frac{1}{3}n + 4$

2. *Given the general term, calculate the value of the term indicated:*

a) $T_n = 4n + 1$, find T_{12} b) $T_n = 3n + \frac{1}{2}$, find T_4

Example: Given the following sequence, find the formula:

3; 6; 9; 12; 15; ……..

Therefore:

The common difference is $T_2 - T_1 = 6 - 3 = 3$
Now substitute the first term into *a* and you have your formula:

$\therefore T_n = 3 + (n - 1)(3)$

Activity 2.3 continued

3. *Determine the 15^{th} and 100^{th} term of each of the following arithemetic sequences by first finding T_n:*

a) 9; 12; 15; … b) $0; -\frac{5}{2}; -5; \ldots$

c) $7 + 9x; 8 + 11x; 9 + 13x$ d) $a = 5; d = 7x$

4. *Determine which term of the arithmetic sequence is equal to the term given in brackets:*

a) -2; 1; 4… (109) b) $\frac{1}{2}; -\frac{3}{7}; -\frac{19}{14}; \ldots$ (-19)

c) $x; 2x + 3; 3x + 6; \ldots (50x + 147)$

d) $\log\sqrt{2}\ ; \log 4\ ; \log 8\sqrt{2}\ \ldots (\log 2^{23})$

5. *Determine the arithmetic sequence and the 14^{th} term in each of the following:*

a) The 4^{th} term is 14 and the 20^{th} term is 94.

b) The 7^{th} term is 12 and the 33^{rd} term is -40.

c) The 5^{th} term is $2 + 3x$ and $d = 1 + x$

d) The 6^{th} term is $5x - 2$ and $a = -3$

6. Consider the following arithmetic sequence:

$x + 3; 2x - 2\ ; 5x + 1; \ldots$

a) Find the value of x.

b) Write down the first 3 terms.

c) Determine the 20^{th} term of this sequence.

d) Which term in this sequence will be equal -64?

<u>More notes on arithmetic sequences</u>:

An arithmetic sequence, or progression, is any sequence where the same difference occurs between each term within that sequence.

We use the following for the formula: $T_n = a + (n - 1)d$

'a' is the first term

'd' is the common difference between the terms

Activity 2.4

1. *Determine the 20th term of the following arithmetic sequences:*

a) 6; 12; 18;...

b) -11; -9; -7;...

c) p + 2q; 3p + 3q; 5p + 4q;...

d) $a = -\frac{1}{4}$, $d = \frac{3}{4}$

e) a = 2, T_6 = 62 (HINT: Find 'd' first)

2. *For each of the following formulas, find:*

(i) Term 1 (a)

(ii) Common difference (d)

a) $T_n = 4n - 2$

b) $T_n = 5 + 3n$

c) $T_n = \frac{1}{2}n$

d) $T_n = 6-2n$

3. *Determine which term in each of the below sequences:*

a) 2; 4; 6; … is equal to 48

b) -3; 1; 5; … is equal to 81

c) $2\frac{1}{4}; 2\frac{1}{2}; 2\frac{3}{4}$; is equal to $6\frac{3}{4}$

d) a; 2a + 1; 3a + 2; … is equal to 12a + 11

4. *Given the following terms of an arithmetic sequence, determine the first 3 terms, and then the value of T_{25}:*

a) T_8= 22, and T_{17}= 49

b) T_{11}= -28, and T_{30}= -104

c) T_6= 10, and T_{21}= $14\frac{1}{2}$

d) T_4= 7x + 10, and T_{15}= 29x +43

D. Geometric Sequences

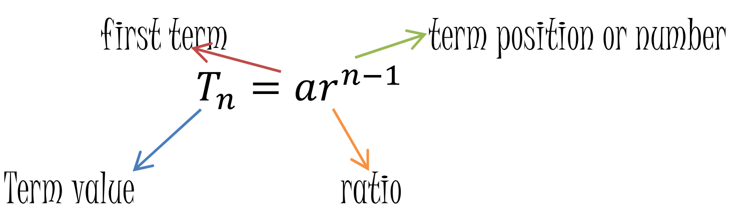
Notes:

In a geometric sequence, there is a common ratio, ‘r’, which is calculated by dividing any term of a sequence by the previous term in the sequence. Therefore:

$\frac{T_{n+1}}{T_n} = r$, for n ≥ 1 this can also be used to prove that a sequence is geometric:

If $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ then the sequence has a common ratio and ∴ the sequence is geometric.

The formula for finding the general term in a geometric sequence is:



Example 1:

For the sequence given below:

a) show that it is a geometric sequence, and

b) calculate the values for ‘a’ and ‘r’ to find the general term, in order to calculate the next three terms of the sequence.

72; 12; 2; .....

Answer:

a) $T_1 = 72$ Therefore, $\frac{T_2}{T_1} = \frac{12}{72} = \frac{1}{6}$ and $\frac{T_3}{T_2} = \frac{2}{12} = \frac{1}{6}$

$T_2 = 12$ There is a common ratio of $\frac{1}{6}$ and a = 72

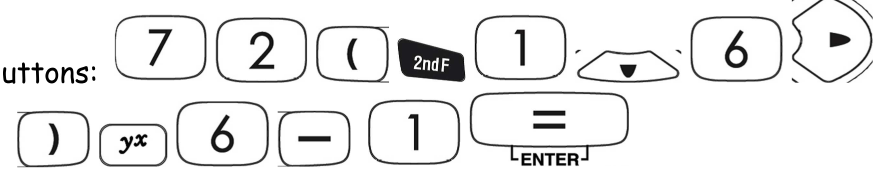
$T_3 = 2$ The general term is: $T_n = 72.\left(\frac{1}{6}\right)^{n-1}$

The next three terms of the sequence are:

$T_4 = 72.\left(\frac{1}{6}\right)^{4-1}$, $T_5 = 72.\left(\frac{1}{6}\right)^{5-1}$, $T_6 = 72.\left(\frac{1}{6}\right)^{6-1}$

$= \frac{1}{3}$ $= \frac{1}{18}$ $= \frac{1}{108}$

All of these can be entered straight into your SHARP EL-W535HT calculator by pressing these buttons:



Example 2:

Given the following geometric sequence, calculate the 10^{th} and 21^{st} terms:

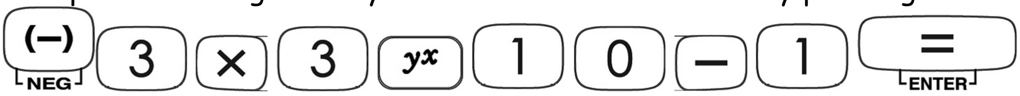
-3;   -9;   -27; ….

Answer:

a = -3,   r = 3 (remember $\rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2}$)

$T_{10} = -3 \times (3)^{10-1} = -59\,049 \; or -3 \times 3^9$, and
$T_{21} = -3 \times (3)^{21-1} = -1.046 \times 10^{10} \; or -3 \times 3^{20}$

You can also put this straight into your EL-W535HT calculator by pressing these buttons:

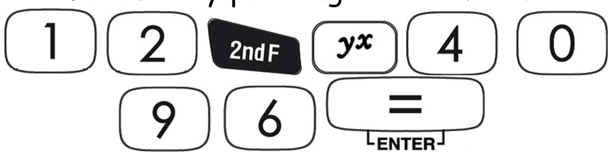


Example 3:

Given that $T_3 = 8$ and $T_{15} = 32\,768$, determine the first three terms of the geometric sequence.

$T_3 = ar^{3-1} = 8$ (1)
$T_{15} = ar^{15-1} = 32\,768$ (2)
Therefore: $\frac{ar^{14}}{ar^2} = \frac{32\,768}{8}$
$r^{12} = 4\,096$
$\therefore r = \sqrt[12]{4\,096}$
$\therefore r = 2$

Again you can simply put this into your EL-W535HT by pressing these buttons:



<u>Substitute r = 2 into (1):</u> $8 = a.(2)^2$
8 = 4a
Therefore: 2 = a

$T_1 = 2; \;\; T_2 = 2 \times 2 = 4; \;\; T_3 = 2 \times 2^2 = 8$

Example 4:

In the following geometric sequence $\frac{1}{3}$; $\frac{1}{15}$; $\frac{1}{75}$ … Which term in this sequence is equal to $\frac{1}{9375}$?

$a = \frac{1}{3}$

$r = \frac{\frac{1}{15}}{\frac{1}{3}} = \frac{1}{5}$

$T_n = \frac{1}{3} \times \left(\frac{1}{5}\right)^{n-1} = \frac{1}{9375}$

$\therefore \left(\frac{1}{5}\right)^{n-1} = \frac{1}{3125}$

$\therefore n - 1 = \log_{\frac{1}{5}} \frac{1}{3125}$

$\therefore n - 1 = 5$

$\therefore n = 6$

Example 5:

Given that $T_7 = 360$ and a common ratio of 3, determine the first three terms of the geometric sequence.

$T_7 = a.r^{7-1} = 360$

$a.3^6 = 360$

$a.729 = 360$

$a = \frac{360}{729}$

$a = \frac{40}{81}$

$\therefore\ T_1 = \frac{40}{81}\ ;\quad T_2 = \frac{40}{27}\ ;\quad T_3 = \frac{40}{9}$

Activity 2.5

1. *Say whether the following sequences are geometric or arithmetic. Find the next three terms in each of the sequences:*

a) $\frac{1}{4}$; $\frac{1}{10}$; $\frac{1}{25}$

b) 3; 12; 48

c) -7; -10; -13

d) 5; 9; 13

e) $-\frac{3}{4}$; $-\frac{9}{8}$; $-\frac{27}{16}$

f) $\frac{2}{7}$; $\frac{11}{14}$; $\frac{9}{7}$

2. *From the given terms, calculate the common ratio, and then calculate the 21st term of the geometric sequence:*

a) $T_1 = 2$ and $T_{12} = \frac{1}{1024}$

b) $T_1 = \frac{1}{2}$ and $T_{12} = -88\,573\frac{1}{2}$

c) $T_1 = 4x$ and $T_{12} = 8192x^{12}$

d) $T_1 = 8$ and $T_{12} = \frac{177\,147}{256}$

3. *Given 'a' and 'r', find the first three terms and also find out which term is equal to the value shown below:*

a) $a = 1, r = 2, T_n = 32\,768$

b) $a = \frac{1}{2}, r = 3, T_n = 3280\,\frac{1}{2}$

c) $a = \frac{1}{16}, r = 2, T_n = 4096$

d) $a = 8, r = \frac{1}{4}, T_n = \frac{1}{32}$

4. *Calculate 'r', where 'a' is given along with a term total:*

a) $a = 2\ and\ T_5 = 32$

b) $a = 1\ and\ T_7 = 729$

c) $a = \frac{1}{3}\ and\ T_6 = 2\,592$

d) $a = -5\ and\ T_9 = -\frac{5}{256}$

5. In a geometric sequence, the first three terms are given as:
(p + 2); (p - 2); and p. Find the value of p, and hence the first three terms.

E. Series

- A sequence is an ordered list, i.e. $T_1; T_2; T_3; \ldots\ldots\ldots; T_n$
- A series is the sum of the terms of the sequence, i.e. $T_1 + T_2 + T_3 + \ldots T_n$
- A finite series is the sum of a given number of terms, whereas an infinite series is the sum of all the terms of a sequence. An infinite series only occurs when a series is geometric and $-1 < r < 1; r \neq 0$ (or in other words your ratio is a positive or negative fraction). This concept will be explored in more detail in Section F page 55.
- The formula for the sum of 'n' terms of an arithmetic series: $S_n = \frac{n}{2}[2a + (n-1)d]$
- If the last term of the sequence is given, then the formula is as follows: $S_n = \frac{n}{2}[a + l], l = a + (n-1)d$
- $S_n = S_{n-1} + T_n, \therefore T_n = S_n - S_{n-1}$
- The formula for the sum of a geometric series to n terms is: $S_n = \frac{a(1-r^n)}{1-r}$ or $S_n = \frac{a(r^n-1)}{r^n-1}$

As a matric student you need to be able to prove these two series (sum) formulae. Here is how to do it:

$S_n = a + [a + d] + [a + 2d] + [a + 3d] + \ldots.. + [a + (n - 2)d] + [a + (n-1)d]$
$S_n = [a + (n-1)d] + [a + (n-2)d] + [a + (n-3)d] + \ldots. + [a + d] + a$

Add these two sums together:
$2S_n = [2a + (n-1)d] + [2a + (n-1)d] + [2a + (n-1)d] + \ldots + [2a + (n-1)d]$
$= n[2a + (n-1)d]$

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

And for the geometric series the proof is just as easy:

$S_n = a + ar + ar^2 + ar^3 + \ldots. + ar^{n-2} + ar^{n-1}$
$r \times S_n = ar + ar^2 + ar^3 + \ldots. + ar^{n-1} + ar^n$ (Multiply by r)
$rS_n - S_n = -a + 0 + 0 + 0 + \ldots. + 0 + 0 + ar^n$

$$\therefore r S_n - S_n = ar^n - a$$
$$\therefore S_n(r-1) = a(r^n - 1)$$
$$\therefore S_n = \frac{a(r^n-1)}{(r-1)}, r \neq 1.$$

Example 1:

If T_1 = 2 and d = 5,

a) Determine T_1 to T_5 of the sequence and

b) Calculate the sum of these first five terms.

Answers

a) T_1 = 2, T_2 = 7, T_3 = 12, T_4 = 17, T_5 = 22

b) $S_n = \frac{n}{2}[a + l]$

$S_5 = \frac{5}{2}[2 + 22]$

$S_5 = 60$

Example 2:

Given that $S_n = \frac{1}{4}n^2 - n$ find the first 5 terms and say whether the sequence is arithmetic or geometric, and then calculate the sum of these 5 terms.

$S_1 = \frac{1}{4}(1)^2 - (1) = -\frac{3}{4}$ $\therefore T_1 = S_1 = -\frac{3}{4}$

$S_2 = \frac{1}{4}(2)^2 - (2) = -1$ $\therefore T_2 = S_2 - S_1 = -1 - \left(-\frac{3}{4}\right) = -\frac{1}{4}$ $S_3 =$

$\frac{1}{4}(3)^2 - (3) = -\frac{3}{4}$ $\therefore T_3 = S_3 - S_2 = \left(-\frac{3}{4}\right) - (-1) = \frac{1}{4}$

$S_4 = \frac{1}{4}(4)^2 - (4) = 0$ $\therefore T_4 = S_4 - S_3 = 0 - (-\frac{3}{4}) = \frac{3}{4}$

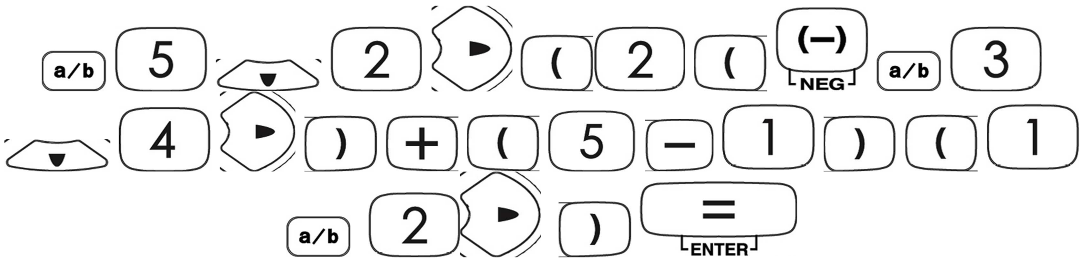
$S_5 = \frac{1}{4}(5)^2 - (5) = 1\frac{1}{4}$ $\therefore T_5 = S_5 - S_4 = 1\frac{1}{4} - 0 = 1\frac{1}{4}$

$d = \frac{1}{2}$ so this is an arithmetic sequence as there is a common difference.

$\therefore S_5 = \frac{5}{2}\left[2\left(-\frac{3}{4}\right) + (5-1)\left(\frac{1}{2}\right)\right]$

$S_5 = 1\frac{1}{4}$

You can put this sequence straight into your SHARP EL-W535 calculator and get the correct answer:



Example 3:

Given that T_1 = 0,1 and T_{15} = 2,9, calculate the sum of the arithmetic series to 15 terms.

T_1 = 0,1 = a

$T_{15} = a + (n-1)d = 2.9$

$0.1 + (14)d = 2.9$

$14d = 2.8$

$d = \frac{1}{5}\ or\ 0.2$

$\therefore\ S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore\ S_{15} = \frac{15}{2}\left[2(0.1) + 14\left(\frac{1}{5}\right)\right]$

$\therefore\ S_{15} = 22\frac{1}{2}\ or\ 22.5$

Example 4:

Given an arithmetic sequence of 32 +28 + 24 + …… Determine the value of 'n' for which the series total is 140.

$\therefore\ S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore 140 = \frac{n}{2}[2(32) + (n-1)(-4)]$

$\therefore 280 = n[64 - 4n + 4]$

$\therefore 280 = 64n - 4n^2 + 4n$

$\therefore 0 = 4n^2 - 68n + 280$

$\therefore 0 = n^2 - 17n + 70$

$\therefore 0 = (n-10)(n-7)$

n = 7 or n = 10.

Both values are positive and if they are substituted into the formula both give a result of 140, so they both satisfy the equation.

Example 5:

The sum of the first 10 terms of an arithmetic series is 80. The sum of term 3 and term 7 is 12. Calculate 'a' and 'd', and hence write down the series.

$T_3 = a + 2d$

$T_7 = a + 6d$

$\therefore a + 2d + a + 6d = 12$

$\therefore 2a = 12 - 8d$

Remember that 2a = 12 – 8d

$\therefore\ S_{10} = 80 = \frac{10}{2}[2a + 9d]$

$\therefore 80 = 5[12 - 8d + 9d]$

$80 = 5[12 + d]$

$\frac{80}{5} = 12 + d$

$16 - 12 = d$

$d = 4$

$\therefore 2a = 12 - 8d$
$\therefore a = 6 - 4d$
$\therefore a = 6 - 4(4)$
$\therefore a = -10$

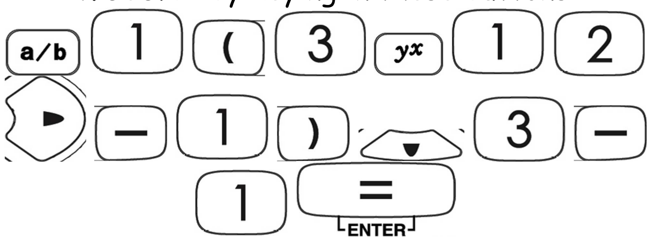
$$\therefore -10 \; -6 \; -2 + \; 2 + \; 6 + \cdots$$

Example 6:

Calculate the geometric series: 1+3+9+27…..to 12 terms.

$S_n = \frac{a(1-r^n)}{1-r}$
$\therefore S_{12} = \frac{1(1-3^{12})}{1-3}$
$\therefore S_{12} = 265\,720$

You can put this straight into your SHARP EL-W535HT by keying in these buttons:



Example 7:

Calculate the value of 'n', for which the series is equal to $16\,383\frac{1}{2}$, and where term 1 is $\frac{1}{2}$ and r = 2.

$S_n = \frac{a(1-r^n)}{1-r}$
$\therefore 16\,383\frac{1}{2} = \frac{\frac{1}{2}(1-2^n)}{1-2}$
$\therefore -\frac{32\,767}{2} = \frac{1}{2}(1-2^n)$
$\therefore -32\,767 = 1 - 2^n$
$\therefore 32\,768 = 2^n$
$\therefore \log_2 32\,768 = n$
$\therefore n = 15$

Activity 2.6

1. *Given the following general terms, determine the first 3 terms of the sequence, and hence calculate the sum of those three terms.*
   a) $T_n = -2n$ b) $T_n = 4 - n$

2. Given the general term, $T_n = 2n + 3$, calculate the sum of the first 10 terms, i.e. S_{10}.

3. If S_{10} = 120 and S_{11} = 144, find the value of T_{11}.

4. If $S_n = 3n^2 + n$, find the first 5 terms of the sequence, and say what type of sequence it is.

5. Given the following arithmetic series , find the sum: (Hint find n first)

a) 3 + 6 + 9 + 12+…… +36.
b) $2 + 2\frac{1}{2} + 3 + 3\frac{1}{2} +$ …. 10.
c) 7 + 6,75 + 6,5 + 6,25 + …… 3,75.

6. Find the number of terms in the arithmetic series if the sum is 45, $T_2 = \frac{3}{2}$ and T_5 = 3.

7. S_6 = 159, $T_8 - T_5$ = 15. Find S_{10}.

8. *Calculate the sum of each geometric series given below:*

a) 8 + 4 + 2 + ……… to 10 terms
b) 2 + -4 + 8 + ……… to 6 terms
c) $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} +$ … .. to 8 terms.

9. *If:*
a) T_1 = 3 and $T_7 = \frac{3}{64}$. Find S_{10} of the geometric series.
b) T_1 = 2 and T_8 = 4 374. Find S_{15} of the geometric seies.

10. S_6 = 504 and r = 2. Find T_1.

F. Sigma Notation

$$\sum_{n=1}^{k} T_n$$

- $\sum$ →This symbol is the Greek capital letter for S. It represents the sum of a number of terms in a sequence. The above example reads as follows: Sigma T_n, starting at n = 1, and ending with k = what?

(the sum of a number of terms of a general term)

There are certain sigma series that have a particular answer:

- $\sum_{n=1}^{k} 1 = n$ in other words → 1 + 1 + 1 + 1 and so on…. This is simply counting and so will equal the number of terms in the sequence.
- $\sum_{n=1}^{k} n = \frac{k(k+1)}{2}$ in other words 1 + 2 + 3 + 4 and so on… if you use the formula you should find the sum.

  For example if you have: $\sum_{n=1}^{5} n$ then you are saying 1 + 2 + 3 + 4 + 5 = 15. Simple if you have only 5 terms.

  But when you have: $\sum_{n=1}^{100} n$ then it becomes much more difficult and you need to use the formula →$\frac{k\,(k+1)}{2}$

  $\therefore \frac{100(100+1)}{2}$

  (which you can put straight into your EL-W535HT.) $\therefore 5050$

Example 1: $\sum_{n=1}^{8} n(n-1)$

This example is asking that you calculate the sum where 1 is the first value to be substituted and 8 is the last value to be substituted. So in all, 8 terms will be added to get the answer.

Solution: $\sum_{n=1}^{8} n(n-1)$

= 1(0) + 2(1) + 3(2) + 4(3) + 5(4) + 6(5) + 7(6) + 8(7)

= 0 + 2 + 6 + 12 + 20 + 30 + 42 + 56

= 168

Example 2*:* $\sum_{n=4}^{12} 8$

In this example, no general term is given, so that means that each term in the sequence will be the number 8. There will be 9 terms of 8 added together as the starting value is 4 and the ending value is 12

When you want to work out how many terms there are in $\sum_{n=i}^{k} T(n)$
$\rightarrow$ k - n + 1

Solution: $\sum_{n=4}^{12} 8$
= 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8
= 72

OR $= 8 \times 9 = 72$

Example 3: In this example, a series is given, which needs to be converted back to sigma notation.

3 + 6 + 9 + 12 + 15 +18 +21

Convert it to sigma notation

Solution: Try to find the arithmetic or geometric rule for the sequence.

3 + 6 + 9 + 12 + 15 +18 +21

$\therefore a = 3; \quad d = 3$

$\therefore T_n = 3 + (n-1)(3)$

$= 3 + 3n - 3$

$\therefore T_n = 3n$

3 + 6 + 9 + 12 + 15 +18 +21 = $\sum_{n=1}^{7} 3n$

the last term position in the sequence (7)

The equation or formula ($3n$)

What term are you starting at? ($n = 1$)

Example 4:

5 + 9 + 13 + 17….to k terms) is equal to 324. Find k.

Solution:

a = 5, d = 4

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore 324 = \frac{k}{2}[2(5) + (k-1)4]$$

$$\therefore 648 = k[10 + 4k - 4]$$

$$\therefore 0 = 10k + 4k^2 - 4k - 648$$

$$\therefore 0 = 4k^2 + 6k - 648$$

$$\therefore 0 = 2k^2 + 3k - 324$$

$$\therefore 0 = (k-12)(2k+27)$$

$$\therefore k = 12 \quad or\ k \neq -\frac{27}{2}$$

as n ≠ a fraction or a negative number.

Example 5: $\sum_{n=1}^{150}(2n+1)$

If the number of terms given is too large to calculate the sum of, as in the above examples, the formula for arithmetic series or geometric series can be used. First you would need to decide whether the sequence has a common difference or ratio.

Answer:

3 + 5 + 7 + ……. + 301. a = 3, d = 2, therefore this an arithmetic sequence. The last term given is 301.

$\therefore\ S_n = \frac{n}{2}[2a + (n-1)d]$ OR $S_n = \frac{n}{2}(a + l)$

$\therefore\ S_{150} = \frac{150}{2}[2(3) + (150-1)(2)]$

$= 75[6 + 298]$

= 75 (304)

= 22 800

$\therefore S_{150} = \frac{150}{2}(3 + 301)$

$= 22\ 800$

> The general term for a geometric sequence is $T_n = ar^{n-1}$, and the equation for the sum of a geometric series is $S_n = \frac{a(1-r^n)}{1-r}$, r $\neq$ 1. In sigma notation this would read as $S_n = \sum_{n=1}^{k} ar^{n-1}$.

Example 6:

Given:

$$S_{100} = \sum_{n=1}^{100} \frac{1}{2}(2)^{n-1}$$

Find the sum of the first 100 terms of the geometric sequence.

Therefore:

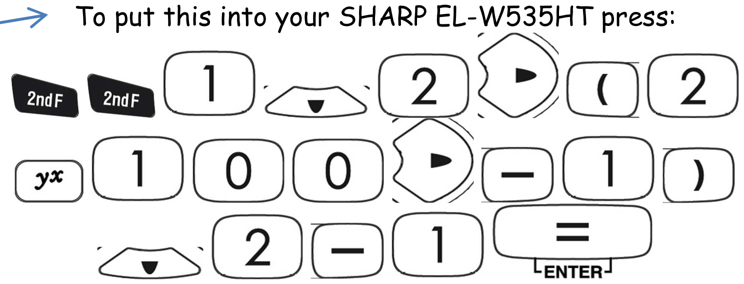
$a = \frac{1}{2}(2)^0 = \frac{1}{2};$ $T_2 = \frac{1}{2}(2)^1 = 1;$ $T_3 = \frac{1}{2}(2)^2 = 2;$ $\therefore r = 2$

$\therefore\ S_n = \frac{a(1-r^n)}{1-r}$

$\therefore\ S_{100} = \frac{\frac{1}{2}(2^{100}-1)}{2-1}$

$\therefore\ S_{100} = 6.338 \times 10^{29}$

To put this into your SHARP EL-W535HT press:



Example 7:

Given:

1 + 3 + 9 + 27 + …… to 12 terms. Calculate the sum of this geometric series.

Answer:

$a = 1$

$r = 3$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{12} = \frac{1(1-3^{12})}{1-3}$$

$$\therefore S_{12} = 265\ 720$$

<u>Activity 2.7</u>

1. *Evaluate (means calculate) the following series by first determining if the series is arithmetic or geometric:*

    a) 2 – 6 – 18………. To 8 terms.
    b) 3 + 6 + 9………. To 8 terms.
    c) ½ + 1+ 2………. To 8 terms.
    d) 16 + 12 + 8………. To 8 terms.

2. *Determine n in each of the following series by first identifying whether it is arithmetic or geometric:*

    a) $\sum_{k=1}^{n} \left(\frac{1}{2}\right)(3)^{k-1} = 4920\frac{1}{2}$
    b) $\sum_{k=1}^{n} (2k+3) = 285$
    c) $\sum_{k=1}^{n} (5)(3)^{k-1} = 16400$

3. *Calculate:*

    a) $\sum_{n=1}^{5} \left(\frac{1}{2}\right)(4)^{n-1}$
    b) $\sum_{n=3}^{8} \left(\frac{5n}{2}\right)$
    c) $\sum_{n=6}^{18} 3.2n$
    d) $\sum_{n=2}^{6} (4)\left(\frac{1}{2}\right)^{n-1}$

4. *Given the following information, find the value of 'n', or 'k':*

    a) S_n = 182, a = 8, and d = 6
    b) S_n = 3 069, a = 3 and r = 2.

5. *Evaluate the following:*

a) $\sum_{n=1}^{100}(3n-5)$ b) $\sum_{n=1}^{150}\left(\frac{1}{2}\right)(3)^{n-1}$

6. *Write the following in sigma notation; all to k terms.*

a) - 1 – 3 – 9………………
b) 2 + 1 + ½ ………………..

G. Sum to Infinity

When no last term of a series is given, we cannot calculate a definite sum. We say we are finding the sum to infinity. This is symbolised by using ∞.
In an infinite arithmetic series, the larger the value for 'n', the larger the answer. That is, this series will always diverge → in other words, the answer for a sum to infinity of a divergent series is infinity.
In an infinite geometric series, the result can either be divergent or convergent, depending on the size of 'r'. For the series to be divergent, the value for 'r' must be less than -1, r = 1, or r is greater than 1. For a convergent series, 'r' must lie between -1 and 1 (remember that r ≠ 0).

$$S_\infty = \frac{a}{1-r} \quad or \quad \sum_{n=1}^{\infty}(a)(r)^{n-1} = \frac{a}{1-r} \qquad because\ r^n \to 0 \quad as \quad n \to \infty$$

Example 1:

Find S_∞ for the following sequence. Give a reason for the existence or non-existence of S_∞.

$$\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$$

$\therefore\ T_1 = \frac{1}{2}; \qquad T_2 = \frac{1}{4}; \qquad T_3 = \frac{1}{8}$

$\therefore a = \frac{1}{2} \quad and \quad r = \frac{1}{2}$

$\therefore\ S_\infty = \frac{a}{1-r}$

$\therefore\ S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{2}}$

$\therefore\ S_\infty = 1$

∴ The sum to infinity exists as -1 < r < 1 (the sum converges to 1 as 'n' increases).

Example 2:

Calculate the following if it exists:

$$\sum_{n=1}^{\infty}(2)\left(\frac{3}{2}\right)^{n-1}$$

$\therefore a = 2 \quad and \quad r = \frac{3}{2}$

$\therefore$ S_∞ cannot be calculated.

The sum of this series does not exist as r > 1

Example 3:

If S_∞ is equal to $10\frac{2}{3}$ and a = 8, find the value of 'r':

$\therefore \ S_\infty = \frac{a}{1-r}$

$\therefore 10\frac{2}{3} = \frac{8}{1-r}$ Change $10\frac{2}{3}$ to an improper fraction

$\therefore \ \frac{32}{3} = \frac{8}{1-r}$ Cross multiply

$\therefore 32 - 32r = 24$

$\therefore \ -32r = -8$

$\therefore r = \frac{1}{4}$

Example 4:

Given the series: (p+2) + $(p+2)^2$ + $(p+2)^3$ .....

Determine for which values of 'p' the series will converge, and hence find the sum of the series in terms of *p*. (For a converging series, -1 < r < 1)

a = (p+2) Therefore: -1 < p+2 < 1

r = (p+2) -3 < p < -1 (subtract 2 from both sides)

So for -3 < p < -1, $S_\infty = \frac{p+2}{1-(p+2)}$

$= \frac{p+2}{-1-p}$ for a convergent series

Activity 2.8

1. *Given the following geometric series, find the sum to infinity*:

a) $\sum_{n=1}^{\infty}(2)\left(\frac{1}{3}\right)^{n-1}$

b) 5,25 + 0,0525 + 0,000525

c) $1 + \frac{1}{3} + \frac{1}{9} \ldots$

d) $\sum_{p=0}^{\infty}\left(-\frac{1}{2}\right)^{p}$

2. *Determine whether it is possible to calculate the sum to infinity:*

$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)(2)^{n-1}$

3. $If\ S_{\infty} = 15, \quad a = 3. Find\ r.$

4. $If\ S_{\infty} = \frac{7}{16}, \quad r = \frac{1}{8}. Find\ a.$

5. A mountain climber is climbing a mountain. In his first hour he climbs 2km. For every hour he climbs thereafter, he is only able to climb $\frac{3}{5}$ of the previous height, due to an increase in the incline of the mountain. How high is he able to climb before he needs to start using climbing equipment?

H. Problem solving

Steps to solving problems:

- Know what you are being asked to find
- Write down information that will be useful in solving the problem
- Use diagrams to help understand the information in the question
- Make sure that all units given are the same, otherwise convert
- Use correct formulae to assign variables
- If solving a sequence problem, make sure you identify whether it is arithmetic or geometric
- Solve for the unknown value by using appropriate equations
- Make sure the answer is logical and correct. Substitute it back to check the accuracy.

Example:

Matchsticks are arranged in piles. The first pile consists of 10 matchsticks. Each pile thereafter consists of 8 matchsticks more than the previous pile.

How many matchsticks are in the 25^{th} pile?
If a pile consists of 98 matches, work out which pile it is.
Work out how many matches there are in the first 40 piles.

Solutions:

a) This sequence is arithmetic. Therefore, $T_{25} = 10 + (24)(8) = 202$
There are 202 matches in the 25^{th} pile.

b) $T_n = 10 + (n-1)8 = 98$
$8n - 8 = 88$
$8n = 96$
$n = 12$

c) $S_{40} = \frac{40}{2}[2(10) + (40-1)8]$
$= 6640$

Activity 2.9

1. *On a certain day, 2 learners are found to be infected with the flu virus. Each day thereafter, the number of new learners infected is three times the previous day's total*.

   a) Calculate how many people become infected on the 5th day.
   b) If there are 2186 learners enrolled at the school, assuming that no one is absent, how long would it take for all the learners to become infected?
   c) Calculate the total number of learners that are infected after 3 days of school.

2. *A census is done on a city and it is found that there is a population of 5000 people. Every successive year, the population increases by a tenth of the previous year's total population.*

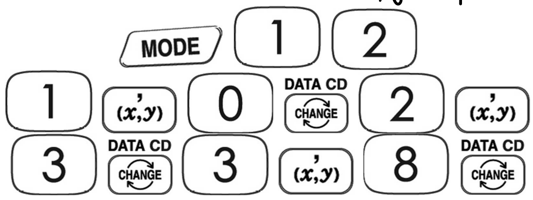
   a) How many people are there in the third year after the census was completed?
   b) In what year after the census did the population reach 15000?
   c) How many people are found to be living in this city after 10 years?

Answers for Activities

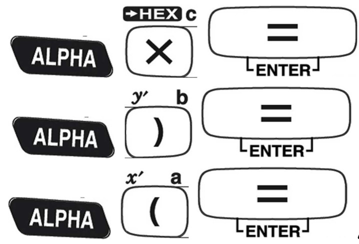
Activity 2.1

1. a) Formula: $T_n = x^2 - 1$
   Next three terms: 24; 35; 48

You can use the quadratic function in stats mode to work this out, just press:



Now to find $cn^2 + bn + a$ press:



Ans: 1

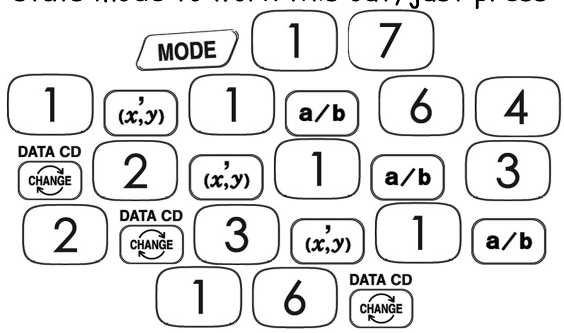
Ans: 0

Ans: -1

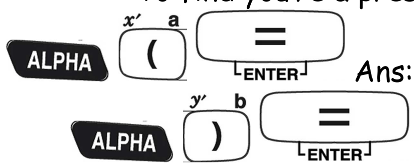
b) Formula: terms 1,3 and 5 are increasing negative odd numbers, terms 2,4 and 6 are increasing positive odd numbers
   Next three terms: -7; 7; -9.

c) Formula: multiplying each new term by 2 so: $T_n = \left(\frac{1}{64}\right) 2^{n-1}$
   Next three terms: $\frac{1}{8}$; $\frac{1}{4}$; $\frac{1}{2}$

You can use the exponential function in stats mode to work this out, just press:



To find you're a press:



Ans: 0,0078125

Ans: 2

d) Formula: Increasing order of 3^{n-1}
   Next three terms: 81; 243; 729

e) Formula: adding alternatively either a 1 or 0 after the decimal point.
   Next three terms: 0.10101; 0.010101; 0,1010101

f) Formula: Multiplying the next term by $2xy^2$
   Next three terms: $8x^5y^7$; $16x^6y^9$; $32x^7y^{11}$

Activity 2.2

1. a)

2 5 10 17 26

3 5 7 9

2 2 2

a + b + c = 2

3a + b = 3

2a = 2

a = 1

3(1) + b = 3

b = 0

1 + 0 + c = 2

c = 1

$T_n = n^2 + 1$

OR $T_n = T_1 + (n-1)f + \frac{(n-1)(n-2)s}{2}$

$\therefore T_n = 2 + (n-1)(3) + \frac{(n-1)(n-2)(2)}{2}$

$\therefore T_n = 2 + 3n - 3 + n^2 - 3n + 2$

$\therefore T_n = n^2 + 1$

b)

1 2 7 16 29 46

1 5 9 13 17

4 4 4 4

a + b + c = 1

3a + b = 1

2a = 4

a = 2

3(2) + b = 1

b = - 5

2 – 5 + c = 1

c = 4

$T_n = 2n^2 - 5n + 4$

OR $T_n = T_1 + (n-1)f + \frac{(n-1)(n-2)s}{2}$

$\therefore T_n = 1 + (n-1)(1) + \frac{(n-1)(n-2)(4)}{2}$

$\therefore T_n = 1 + n - 1 + 2n^2 - 6n + 4$

$\therefore T_n = 2n^2 - 5n + 4$

c)

3 6 12 21 33

3 6 9 12

3 3 3

a + b + c = 3

3a + b = 3

2a = 3

a = $\frac{3}{2}$

$3\left(\frac{3}{2}\right) + b = 3$

b = $-\frac{3}{2}$

$\frac{3}{2} + -\frac{3}{2} + c = 3$

c = 3

$T_n = \frac{3}{2}n^2 - \frac{3}{2}n + 3$

d) 1   3   10   22   39   61

2   7   12   17   22

5   5   5   5   5

a + b + c = 1 $3\left(\frac{5}{2}\right) + b = 2$

3a + b = 2   b = $-\frac{11}{2}$

2a = 5 $\frac{5}{2} - \frac{11}{2} + c = 1$

a = $\frac{5}{2}$ c = 4 $T_n = \frac{5}{2}n^2 - \frac{11}{2}n + 4$

e) 3   5   8   12   17

2   3   4   5

1   1   1

a + b + c = 3 $3\left(\frac{1}{2}\right) + b = 2$

3a + b = 2   b = $\frac{1}{2}$

2a = 1 $\frac{1}{2} + \frac{1}{2} + c = 3$

a = $\frac{1}{2}$ c = 2

$T_n = \frac{1}{2}n^2 + \frac{1}{2}n + 2$

f) 2   6   11   17   24   32

4   5   6   7   8

1   1   1   1

a + b + c = 2 $3\left(\frac{1}{2}\right) + b = 4$

3a + b = 4   b = $\frac{5}{2}$

2a = 1 $\frac{1}{2} + \frac{5}{2} + c = 2$

a = $\frac{1}{2}$ c = - 1 $\therefore T_n = \frac{1}{2}n^2 + \frac{5}{2}n - 1$

2. 3   4   7   12   19   28

1   3   5   7   9

2   2   2   2   2

a) 28   39   52

9   11   13

2   2

b) $a + b + c = 3$ $\quad 3(1) + b = 1$

$3a + b = 1$ $\quad b = -2$

$2a = 2$ $\quad 1 - 2 + c = 3$

$a = 1$ $\quad c = 4$ $\quad T_n = n^2 - 2n + 4$

c) $T_{22} = (22)^2 - 2(22) + 4$

$= 444$

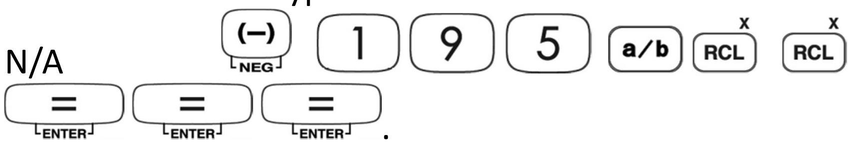
d) $T_n = 199 = n^2 - 2n + 4$

$0 = n^2 - 2n - 195$ [To find the factors go to the table mode

$0 = (n - 15)(n + 13)$ on your calculator (press )

$n = 15$ or $n = -13$ then type in:

N/A



Press the  button and look at the factor pairs for example, does 1 and -195 add up to -2? … No, so we look at the next pair. We continue doing this until we have found both factors, in our example 13 and -15. Once we have our factors we can put them back into the brackets.

3.

2 4 9 17

2 5 8

3 3

a)

28 42 59

11 14 17

3 3

b) $a + b + c = 2$ $\quad 3\left(\frac{3}{2}\right) + b = 2$

$3a + b = 2$ $\quad b = -\frac{5}{2}$

$2a = 3$ $\quad \frac{3}{2} - \frac{5}{2} + c = 2$

$a = \frac{3}{2}$ $\quad c = 3$ $\quad T_n = \frac{3}{2}n^2 - \frac{5}{2}n + 3$

c) $T_{13} = \frac{3}{2}(13)^2 - \frac{5}{2}(13) + 3$

$= 224$

d) $T_n = 612 = \frac{3}{2}n^2 - \frac{5}{2}n + 3$

$1\,224 = 3n^2 - 5n + 6$

$0 = 3n^2 - 5n - 1\,218$

$0 = (3n + 58)(n - 21)$

$n = -\frac{58}{3}$ or $n = 21$

N/A → because n cannot be negative or a fraction.

4. 10 20 36 58

10 16 22

6 6

a) Before you can find n, you need to find the formula:

$a + b + c = 10$ $\quad$ $3(3) + b = 10$

$3a + b = 10$ $\quad$ $b = 1$

$2a = 6$ $\quad$ $3 + 1 + c = 10$

$a = 3$ $\quad$ $c = 6$ $\quad$ $T_n = 3n^2 + n + 6$

$206 = 3n^2 + n + 6$

$0 = 3n^2 + n - 200$

$0 = (3n + 25)(n - 8)$

$n = -\frac{25}{3}$ or $n = 8$

N/A

b) For this question, continue the sequence until your last term (206)

10 20 36 58 86 120 160 206

10 16 22 28 34 40 46

6 6 6 6 6 6

Now add the terms:

10 + 20 + 36 + 58 + 86 + 120 + 160 + 206

= 696 chickens

c) 2 weeks = 14 days n = 14

$T_{14} = 3(14)^2 + (14) + 6$

= 608 chickens on the 14^{th} day.

Activity 2.3

1.

a) -2; -1; 0; 1; 2;.... $T_{100} = 97$

b) 6; 5; 4; 3; 2;..... $T_{100} = -93$

> You can use the table function on the SHARP EL-W535HT calculator to find the terms in the pattern.
>
> To do this you press [MODE] [3] then it will ask you "function?" you enter the T_n = formula into the calculator using x in place of n. For example in 1a. you would press [ALPHA] [RCL (x)] [−] [3] [= ENTER] the calculator will then ask you start? You press [1] [= ENTER] then step – you press [1] [= ENTER] the calculator will then give you a table with X and ANS. X is your term position and continues indefinitely and ANS gives you the term value. You can use this function with any of the patterns and you can find any term position by scrolling down the table - press the [▼] key.
>
> You can also enter the position you are looking for by putting as your start.

c) -2; 0; 2; 4; 6.... $T_{100} = 196$

d) 4; 7; 10; 13; 16.... $T_{100} = 301$

e) 1; 0; -1; -2; -3.... $T_{100} = -98$

f) $4\frac{1}{3}$; $4\frac{2}{3}$; 5; $5\frac{1}{3}$; $5\frac{2}{3}$...... $T_{100} = 37\frac{1}{3}$

2 a) $T_{12} = 4(12) + 1 = 49$

> You can find the answer to this by putting the substitution straight into your EL-W535HT by pressing [4] [×] [1] [2] [+] [1] [= ENTER]

b) $T_4 = 3(4) + \frac{1}{2} = 12\frac{1}{2}$

c) $T_{50} = \frac{2(50)+2}{50}$

$= \frac{102}{50}$

$= \frac{51}{25}$

> To put this into your EL-W535HT you press [2ndF] [2] [(] [5] [0] [)] [+] [2] [▼] [5] [0] [= ENTER] if you want the improper fraction simply press the [CHANGE (DATA CD)] button, if you press the button again you will have your answer in decimal form.

3. a) 9; 12; 15...

$T_n = a + (n-1)d$
$= 9 + (n-1)(3)$
$= 9 + 3n - 3$
$\therefore T_n = 3n + 6$

$\therefore T_{15} = 3(15) + 6$
$= 51$

$\therefore T_{100} = 3(100) + 6$
$= 306$

b) 0; $-\frac{5}{2}$; -5 ....

$T_n = 0 + (n-1)\left(-\frac{5}{2}\right)$
$\therefore T_n = -\frac{5}{2}n + \frac{5}{2}$

$\therefore T_{15} = -\frac{5}{2}(15) + \frac{5}{2}$
$= -35$

$\therefore T_{100} = -\frac{5}{2}(100) + \frac{5}{2}$
$= -247\frac{1}{2}$

c) $7 + 9x$; $8 + 11x$; $9 + 13x$ ...

$T_n = 7 + 9x + (n-1)(1 + 2x)$
$= 7 + 9x + n + 2xn + 1 - 2x$
$\therefore T_n = n + 2xn + 7x + 8$

$\therefore T_{15} = 15 + 2x(15) + 7x + 8$
$= 15 + 30x + 7x + 8$
$= 37x + 23$

$\therefore T_{100} = 100 + 2x(100) + 7x + 8$
$= 100 + 200x + 7x + 8$
$= 207x + 108$

d) $a = 5$; $d = 7x$

$T_n = 5 + (n-1)(7x)$
$= 5 + 7xn - 7x$
$\therefore T_n = 7xn - 7x + 5$

$\therefore T_{15} = 7x(15) - 7x + 5$
$= 105x - 7x + 5$
$= 98x + 5$

$\therefore T_{100} = 7x(100) - 7x + 5$
$= 700x - 7x + 5$
$= 693x + 5$

4. a) -2; 1; 4;... (109)

$\therefore 109 = -2 + (n-1)(3)$
$\therefore 111 = 3(n-1)$
$\therefore 37 = n - 1$
$\therefore n = 38$

b) $\frac{1}{2}$; $-\frac{3}{7}$; $-\frac{9}{14}$; .... (-19)

$\therefore -19 = \frac{1}{2} + (n-1)\left(-\frac{13}{14}\right)$
$\therefore -\frac{39}{2} = \left(-\frac{13}{14}\right)(n-1)$
$\therefore 21 = n - 1$
$\therefore n = 22$

c) $x; \quad 2x+3; \quad 3x+6; \dots (50x+147)$

$\therefore 50x+147 = x+(n-1)(x+3)$
$\therefore 49x+147 = (x+3)(n-1)$
$\therefore \frac{49(x+3)}{(x+3)} = n-1$
$\therefore n = 50$

d) $\log\sqrt{2}\ ; \quad \log 4\ ; \quad \log 8\sqrt{2} \dots (\log 2^{23})$
$\therefore \log 2^{\frac{1}{2}}\ ;\ \log 2^{2};\ \ \log 2^{3\frac{1}{2}} \dots (\log 2^{23}\ )$
$\therefore \frac{1}{2}\log 2\ ;\ 2\log 2\ ;\ 3\frac{1}{2}\log 2 \dots (23\log 2)$

$\therefore 23\log 2 = \frac{1}{2}\log 2 + (n-1)\left(\frac{3}{2}\log 2\right)$
$\therefore 22\frac{1}{2}\log 2 = \left(\frac{3}{2}\log 2\right)(n-1)$
$\therefore 15 = n-1$
$\therefore n = 16$

5. a) $T_4 = 14; \quad T_{20} = 94$

$\therefore a+19d = 94$
$-\ \ a+-3d = -14$
$16d = 80$
$\therefore d = 5$

$\therefore a+19(5) = 94$
$\therefore a = -1$

$\therefore -1; \quad 4; \quad 9 \dots$

$T_n = -1+(n-1)(5)$
$= -1+5n-5$
$\therefore T_n = 5n-6$

$\therefore T_{14} = 5(14)-6$
$= 64$

b) $T_7 = 12; \quad T_{33} = -40$

$\therefore a+32d = -40$
$-a+-6d = -12$
$26d = -52$
$\therefore d = -2$

$\therefore 12 = a+6(-2)$
$\therefore a = 24$

$\therefore 24; \quad 22; \quad 20; \dots$

$\therefore T_n = 24+(n-1)(-2)$
$= 24-2n+2$
$\therefore T_n = -2n+26$

$\therefore T_{14} = -2(14)+26$
$= -2$

c) $T_5 = 2 + 3x\,;\quad d = 1 + x$

$\therefore a + 4(1 + x) = 2 + 3x$
$\therefore a + 4 + 4x = 2 + 3x$
$\therefore a = -2 - x$

$\therefore -2 - x;\quad -1;\quad x\ldots$

$\therefore T_n = -2 - x + (n-1)(1+x)$
$= -2 - x + n + nx - 1 - x$
$\therefore T_n = -3 - 2x + n + xn$

$\therefore T_{14} = -3 - 2x + 14 + 14x$
$= 11 + 12x$

d) $T_6 = 5x - 2;\quad a = -3$

$\therefore -3 + 5(d) = 5x - 2$
$\therefore 5d = 5x + 1$
$\therefore d = x + \frac{1}{5}$

$\therefore T_n = -3 + (n-1)\left(x + \frac{1}{5}\right)$
$= -3 + xn - x + \frac{1}{5}n - \frac{1}{5}$
$\therefore T_n = -3\frac{1}{5} + xn - x + \frac{1}{5}n$

$\therefore -3;\quad -2\frac{4}{5} + x\,;\quad -2\frac{3}{5} + 2x;\ \ldots.$

$\therefore T_{14} = -3\frac{1}{5} + xn - x + \frac{1}{5}n$
$= -\frac{2}{5} + 13x$

6. $x + 3;\quad 2x - 2;\quad 5x + 1;\ \ldots$

a) $2x - 2 - (x + 3) = 5x + 1 - (2x - 2)$
$2x - 2 - x - 3 = 5x + 1 - 2x + 2$
$x - 5 = 3x + 3$
$\therefore -2x = 8$
$\therefore x = -4$

b) $-4 + 3;\quad 2(-4) - 2;\quad 5(-4) + 1$
$\therefore -1;\ -10;\ -19$

c) $\therefore T_n = -1 + (n-1)(-9)$
$\therefore T_{20} = -1 + (20-1)(-9)$
$= -172$

d) $-64 = -1 + (n-1)(-9)$
$-63 = -9(n-1)$
$\therefore n - 1 = 7$
$\therefore n = 8$

Activity 2.4

1.

a) $T_{20} = 6 + (19).6 = 120$

b) $T_{20} = -11+(19).2 = 27$

c) $T_{20} = (p+2q) + (19)(2p+q)$

$= p + 2q + 38p + 19q$

$= 39p + 21q$

d) $T_{20} = -\frac{1}{4} + (19).\left(\frac{3}{4}\right)$

$= \frac{56}{4}$

$= 14$

e) (i) $T_6 = 2 + 5d$

$62 = 2 + 5d$

$60 = 5d$

$12 = d$

(ii) $T_{20} = 2 + (19).12$

$= 230$

2.a) $T_n = 4n - 2$

(i) $T_1 = 4(1) - 2 = 2$

$a = 2$

(ii) $T_1 = 2$

$T_2 = 4(2) - 2 = 6$

$T_3 = 4(3) - 2 = 10$

$d = 4$

b) $T_n = 5 + 3n$

(i) $T_1 = 5 + 3(1) = 8$

$a = 8$

(ii) $T_1 = 8$

$T_2 = 5 + 3(2) = 11$

$T_3 = 5 + 3(3) = 14$

$d = 3$

c) $T_n = \frac{1}{2}n$

(i) $T_1 = \frac{1}{2}(1) = \frac{1}{2}$

$a = \frac{1}{2}$

(ii) $T_1 = \frac{1}{2}$

$T_2 = \frac{1}{2}(2) = 1$

$T_3 = \frac{1}{2}(3) = \frac{3}{2}$

$d = \frac{1}{2}$

d) $T_n = 6 - 2n$

(i) $T_1 = 6 - 2(1) = 4$

$a = 4$

(ii) $T_1 = 4$

$T_2 = 6 - 2(2) = 2$

$T_3 = 6 - 2(3) = 0$

$d = -2$

3.

a) $a = 2, d = 2$

$T_n = 2 + (n - 1).2$

$48 = 2 + 2n - 2$

$48 = 2n$

$24 = n$

$T_{24} = 48$

b) $a = -3, d = 4$

$T_n = -3 + (n - 1).4$

$81 = -3 + 4n - 4$

$88 = 4n$

$22 = n$

$T_{22} = 81$

c) $a = 2\frac{1}{4}, d = \frac{1}{4}$

$T_n = 2\frac{1}{4} + (n - 1).\left(\frac{1}{4}\right)$

$6\frac{3}{4} = 2\frac{1}{4} + \frac{1}{4}n - \frac{1}{4}$

$4\frac{3}{4} = \frac{1}{4}n$

$\frac{19}{4} = \frac{1}{4}n$

$19 = n$ (x4)

$T_{19} = 6\frac{3}{4}$

d) $a = a, d = (a + 1)$

$T_n = a + (n - 1)(a + 1)$

$12a + 11 = a + an + n - a - 1$

$12a + 12 = an + n$

$12(a + 1) = n(a + 1)$

$12 = n\ [(a + 1)]$

$T_{12} = 12a + 11$

4.

a) $T_8 = a + 7d = 22$ (1)

$T_{17} = a + 16d = 49$ (2)

Subtract (1) from (2):

$9d = 27$

$d = 3$

Substitute $d = 3$, into (1):

$22 = a + 7(3)$

$1 = a$

$T_1 = 1, T_2 = 4, T_3 = 7$

$T_{25} = 1 + 24.3 = 73$

b) $T_{11} = a + 10d = -28$ (1)

$T_{30} = a + 29d = -104$ (2)

Subtract (1) from (2):

$19d = -76$

$d = -4$

Substitute $d = -4$, into (1):

$-28 = a + 10(-4)$

$12 = a$

$T_1 = 12, T_2 = 8, T_3 = 4$

$T_{25} = 12 + 24.-4 = -84$

c) $T_6 = a + 5d = 10$ (1)

$T_{21} = a + 20d = 14\frac{1}{2}$ (2)

Subtract (1) from (2):

$15d = \frac{9}{2}$

$d = \frac{3}{10}$

Substitute $d = \frac{3}{10}$ into (1):

$10 = a + 5(\frac{3}{10})$

$10 - \frac{3}{2} = a$

d) $T_4 = a + 3d = 7x + 10$ (1)

$T_{15} = a + 14d = 29x + 43$ (2)

Subtract (1) from (2):

$11d = 22x + 33$

$11d = 11(2x + 3)$

$d = 2x + 3$

Substitute $d = 2x + 3$ into (1):

$7x + 10 = a + 3(2x + 3)$

$\frac{17}{2}$ = a

$T_1 = \frac{17}{2}$, $T_2 = \frac{44}{5}$, $T_3 = \frac{91}{10}$

$T_{25} = \frac{17}{2} + 24.\left(\frac{3}{10}\right)$

$= \frac{157}{10}$

7x + 10 = a + 6x + 9

x + 1 = a

T_1= x + 1, T_2= 3x + 4, T_3= 5x + 7

T_{25} = x + 1 + 24.(2x + 3)

= x + 1 + 48x + 72

= 49x + 73

Activity 2.5

1.

a) $a = \frac{1}{4}$ and $\frac{T_2}{T_1} = \frac{2}{5}$ and $\frac{T_3}{T_2} = \frac{2}{5}$ therefore this is a geometric sequence as there is a common ratio of $\frac{2}{5}$. $T_4 = \frac{2}{125}$

$T_5 = \frac{4}{625}$; $T_6 = \frac{8}{3125}$

b) a = 3 and $\frac{T_2}{T_1} = 4$, $\frac{T_3}{T_2} = 4$ therefore this is a geometric sequence as there is a common ratio of 4. T_4 = 192, T_5 = 768, T_6 = 3072

c) a = -7 and $T_2 - T_1$ = -3, $T_3 - T_2$ = -3 therefore this is an arithmetic sequence as there is a common difference of -3.
T_4 = -16, T_5 = -19, T_6 = -22

d) a = 5 and $T_2 - T_1$ = 4, $T_3 - T_2$ = 4 therefore this is a arithmetic sequence as there is a common difference of 4. T_4 = 17,
T_5 = 21, T_6 = 25.

e) $a = -\frac{3}{4}$ and $\frac{T_2}{T_1} = \frac{3}{2}$, $\frac{T_3}{T_2} = \frac{3}{2}$, therefore this is a geometric sequence as there is a common ratio of $\frac{3}{2}$. $T_4 = -\frac{81}{32}$;
$T_5 = -\frac{243}{64}$, and $T_6 = -\frac{729}{128}$

f) $a = \frac{2}{7}$ and $T_2 - T_1 = \frac{1}{2}$, $T_3 - T_2 = \frac{1}{2}$, therefore this is an arithmetic sequence as there is a common difference of $\frac{1}{2}$.
$T_4 = \frac{25}{14}$; $T_5 = \frac{16}{7}$; $T_6 = \frac{39}{14}$.

2.

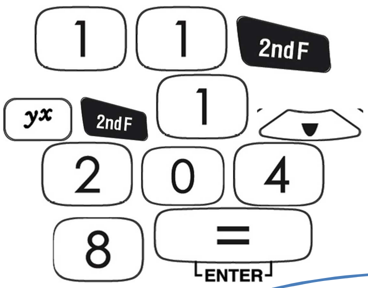
a) $T_{12} = ar^{11} = \frac{1}{1024}$

$2r^{11} = \frac{1}{1024}$

$= \frac{1}{2048}$

$r = \sqrt[11]{\frac{1}{2048}}$

You can solve this by pressing these buttons on your SHARP EL-W535HT calculator:



$T_{21} = 2.\left(\frac{1}{2}\right)^{20}$

$T_{21} = \frac{1}{524\,288}$

b) $T_{12} = ar^{11} = -88\,573\frac{1}{2}$

$\frac{1}{2}r^{11} = \frac{-177\,147}{2}$

$r^{11} = -177\,147$

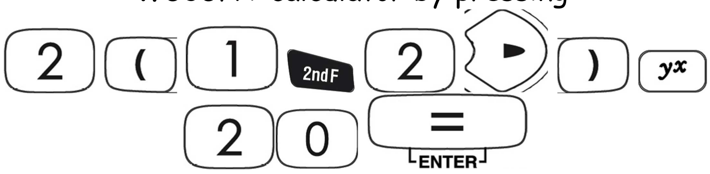
$r = \sqrt[11]{-177\,147}$

$r = -3$

$T_{21} = \frac{1}{2}.(-3)^{20}$

$T_{21} = 1\,743\,392\,201$

Again, you can plug this straight into your SHARP EL-W535HT calculator by pressing:



c) $T_{12} = ar^{11} = 8192x^{12}$

$4xr^{11} = 8192x^{12}$

$r^{11} = 2048x^{11}$

$\sqrt[11]{r^{11}} = \sqrt[11]{(2048x^{11})}$

$r = 2x$

$T_{21} = 4x(2x)^{20}$

$T_{21} = 4\,194\,304x^{21}$

d) $T_{12} = ar^{11} = \frac{177\,147}{256}$

$8r^{11} = \frac{177\,147}{256}$

$\sqrt[11]{r^{11}} = \sqrt[11]{\frac{177\,147}{2048}}$

$r = \frac{3}{2}$

$T_{21} = 8.\left(\frac{3}{2}\right)^{20}$

$T_{21} = 26\,602.05384$

3.

a) $T_n = ar^{n-1}$

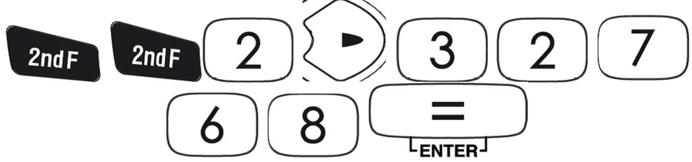
$1 \bullet (2)^{n-1} = 32\,768$

$\log_2 32\,768 = n - 1$

$n - 1 = 15$

$n = 16$

Remember from logs that you can change an exponential equation into a log equation in order to find the exponential unknown. Press the following sequence of keys to get the answer:



b) $T_n = ar^{n-1}$

$-3280\frac{1}{2} = \frac{1}{2}(-3)^{n-1}$

$-6561 = (-3)^{n-1}$

$\log_3 6\,561 = n - 1$

$n - 1 = 8$

$n = 9$

c) $T_n = ar^{n-1}$

$4\,096 = \frac{1}{16}(2)^{n-1}$

$65\,536 = (2)^{n-1}$

$\log_2 65\,536 = n - 1$

$n - 1 = 16$

$n = 17$

d) $T_n = ar^{n-1}$

$\frac{1}{32} = 8(\frac{1}{4})^{n-1}$

$\frac{1}{256} = (\frac{1}{4})^{n-1}$

$\log_{\frac{1}{4}} \frac{1}{256} = n - 1$

$n - 1 = 4$

$n = 5$

4.

a) $T_n = ar^{n-1}$

$T_5 = 2r^4 = 32$

$r^4 = 16$

$\sqrt[4]{r^4} = \sqrt[4]{16}$

$r = 2$

b) $T_n = ar^{n-1}$

$T_7 = r^6 = 729$

$\sqrt[6]{r^6} = \sqrt[6]{729}$

$r = 3$

c) $T_n = ar^{n-1}$

$T_6 = \frac{1}{3}r^5 = 2\,592$

$r^5 = 2592$

$\sqrt[5]{r^5} = \sqrt[5]{7\,776}$

$\therefore r = 6$

d) $T_n = ar^{n-1}$

$T_9 = -5r^8 = -\frac{5}{256}$

$r^8 = \frac{1}{256}$

$\sqrt[8]{r^8} = \sqrt[8]{\frac{1}{256}}$

$\therefore r = \pm\frac{1}{2}$

5. $T_1 = (p+2)$

$T_2 = (p-2)$

$T_3 = p$

$\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\frac{p-2}{p+2} = \frac{p}{p-2}$

$p(p+2) = (p-2)(p-2)$

$p^2 + 2p = p^2 - 4p + 4$

6p = 4

$p = \frac{2}{3} = term\ 3$

$\therefore T_1 = p + 2$

$= \frac{2}{3} + 2$

$= \frac{8}{3}$

$\therefore\ T_2 = p - 2$

$= \frac{2}{3} - 2$

$= -\frac{4}{3}$

<u>Activity 2.6</u>

1.

a) $T_n = -2n \ \rightarrow$ $T_1 = -2(1) T_2 = -2(2)$ $T_3 = -2(3)$

$= -2$ $= -4$ $= -6$

$\therefore\ S_3 = \frac{3}{2}[2(-2) + (3-1)(-2)]$

$= \frac{3}{2}[-8]$

$= -12$

b) $T_n = 4 - n \ \rightarrow$ $T_1 = 4 - 1$ $T_2 = 4 - 2$ $T_3 = 4 - 3$

$= 3$ $= 2$ $= 1$

$\therefore\ S_3 = \frac{3}{2}[2(3) + (3-1)(-1)]$

$= \frac{3}{2}[4]$

$= 6$

2. $T_n = 2n + 3$ $\therefore\ T_1 = 2(1) + 3$ $T_2 = 2(2) + 3$

$= 5$ $= 7$

$T_3 = 2(3) + 3 = 9$ $\therefore a = 5\ and\ d = 2$

$S_{10} = \frac{10}{2}[2(5) + (10-1)(2)]$

$= 140$

3. $S_{10} = 120$ $S_{11} = 144$

$\therefore the\ value\ of\ T_{11}\ is$

$S_{11} - S_{10} = 144 - 120$

$= 24$

4. $S_n = 3n^2 + n$

$$S_1 = 3(1)^2 + 1 = 4 \qquad T_1 = S_1 = 4$$
$$S_2 = 3(2)^2 + 2 = 14 \qquad T_2 = S_2 - S_1 = 14 - 4 = 10$$
$$S_3 = 3(3)^2 + 3 = 30 \qquad T_3 = S_3 - S_2 = 30 - 14 = 16$$
$$S_4 = 3(4)^2 + 4 = 52 \qquad T_4 = S_4 - S_3 = 52 - 30 = 22$$
$$S_5 = 3(5)^2 + 5 = 80 \qquad T_5 = S_5 - S_4 = 80 - 52 = 28$$

There is a common difference of 6, therefore this is an arithmetic sequence.

5.

a) $a = 3, \quad d = 3 \qquad T_n = 36 = 3 + (n-1)(3)$

$$36 = 3 + 3n - 3$$
$$3n = 36$$
$$\therefore n = 12$$

$$S_{12} = \frac{12}{2}[2(3) + (12-1)(3)]$$
$$= 6[39]$$
$$= 234$$

b) $a = 2, \quad d = \frac{1}{2} \qquad T_n = 10 = 2 + (n-1)\left(\frac{1}{2}\right)$

$$10 = 2 + \frac{1}{2}n - \frac{1}{2}$$
$$\frac{1}{2}n = \frac{17}{2}$$
$$n = 17$$

$$S_{17} = \frac{17}{2}\left[2(2) + (17-1)\left(\frac{1}{2}\right)\right]$$
$$= \frac{17}{2}[12]$$
$$= 102$$

c) $a = 7, \quad d = -\frac{1}{4} \qquad T_n = 3.75 = 7 + (n-1)\left(-\frac{1}{4}\right)$

$$3.75 = 7 - \frac{1}{4}n + \frac{1}{4}$$
$$\frac{15}{4} = \frac{29}{4} - \frac{n}{4}$$
$$-\frac{14}{4} = -\frac{n}{4}$$
$$n = 14$$

$$S_{14} = \frac{14}{2}\left[2(7) + (14-1)\left(-\frac{1}{4}\right)\right]$$
$$= 7\left[10\frac{3}{4}\right]$$
$$= 75\frac{1}{4}$$

6. $S_n = 45, \quad T_2 = \frac{3}{2} \quad and \quad T_5 = 3$

$\therefore \frac{3}{2} = a + d$ …… 1

$3 = a + 4d$ …… 2

Equation 2 – Equation 1:

$\frac{3}{2} = 3d$

$\therefore d = \frac{1}{2}$ Substitute into Equation 1

$\therefore \frac{3}{2} = a + \frac{1}{2}$

$\therefore a = 1$

$\therefore S_n = 45 = \frac{n}{2}\left[2(1) + (n-1)\left(\frac{1}{2}\right)\right]$

$90 = n\left[2 + \frac{1}{2}n - \frac{1}{2}\right]$

$90 = 2n + \frac{1}{2}n^2 - \frac{1}{2}n$

$\therefore 0 = n^2 + 3n - 180$

$\therefore 0 = (n+15)(n-12)$

$\therefore n = -15 \quad or\ n = 12$

A term position can never be negative $\therefore n = 12$

7. $T_8 - T_5 = 15$

$\therefore a + 7d - (a + 4d) = 15$

$\therefore 3d = 15$

$\therefore d = 5$

$\therefore S_6 = 159 = \frac{6}{2}[2a + (6-1)(5)]$

$159 = 3[2a + 25]$

$53 = 2a + 25$

$2a = 28$

$\therefore a = 14$

$\therefore S_{10} = \frac{10}{2}[2(14) + (10-1)(5)]$

$\therefore S_{10} = 5[73]$

$\therefore S_{10} = 365$

8.

a) $S_{10} = \frac{8\left(\left(\frac{1}{2}\right)^{10} - 1\right)}{\frac{1}{2} - 1}$

$= 15\,\frac{63}{64}$

b) $S_6 = \frac{2((-2)^6-1)}{-2-1}$
$= \frac{126}{-3}$
$= -42$

c) $S_8 = \frac{\frac{1}{3}\left(\left(\frac{1}{3}\right)^8-1\right)}{\frac{1}{3}-1}$
$= \frac{3280}{6561}$

9.

a) $T_1 = 3 \quad and \quad T_7 = \frac{3}{64}$
$\therefore a = 3$ Substitute into : $T_7 = \frac{3}{64} = ar^6$
$3r^6 = \frac{3}{64}$
$\therefore r^6 = \frac{1}{64}$
$\sqrt[6]{r^6} = \sqrt[6]{\frac{1}{64}} \quad \therefore r = \pm\frac{1}{2}$

$S_{10} = \frac{3\left(\left(\frac{1}{2}\right)^{10}-1\right)}{\frac{1}{2}-1}$ OR $S_{10} = \frac{3\left(\left(-\frac{1}{2}\right)^{10}-1\right)}{-\frac{1}{2}-1}$
$= 5\frac{509}{512}$ $= 1\frac{511}{512}$

b) $T_1 = 2, \quad and \quad T_8 = 4\,374$
$\therefore a = 2$ Substitute into: $T_8 = 4\,374 = ar^7$
$4\,374 = 2r^7$
$\therefore r^7 = 2\,187$
$\sqrt[7]{r^7} = \sqrt[7]{2187}$
$\therefore r = 3$

$S_{15} = \frac{2(3^{15}-1)}{3-1}$

$= 14\,348\,906$

10. $S_6 = 504 = \frac{a(2^6-1)}{2-1}$
$504 = 63a$
$\therefore a = 8$
$\therefore T_1 = 8$

Activity 2.7

1.

a) Geometric sequence: a = -2, r = 3.

$S_8 = \frac{-2(2^6-1)}{2-1}$

$\therefore\ S_8 = -6\ 560$

b) Arithmetic sequence: a = 3, d = 3.

$S_8 = \frac{8}{2}[2(3) + (8-1)(3)]$

$\therefore\ S_8 = 4[27]$

$\therefore\ S_8 = 108$

c) Geometric sequence : a = , r = 2.

$S_8 = \frac{\frac{1}{2}(2^8-1)}{2-1}$

$\therefore\ S_8 = 127\frac{1}{2}$

d) Arithmetic sequence: a = 16, d = -4.

$S_8 = \frac{8}{2}[2(16) + (8-1)(-4)]$

$\therefore\ S_8 = 4[4]$

$\therefore\ S_8 = 16$

2.

a) $4920\frac{1}{2} = \frac{\frac{1}{2}(3^n-1)}{3-1}$

$9841 = \frac{1}{2}(3^n - 1)$

$19\ 682 = 3^n - 1$

$19\ 683 = 3^n$

$\log_3 19\ 683 = n$

$n = 9$

b) $285 = \frac{n}{2}[2(5) + (n-1)(2)]$

$570 = n[10 + 2n - 2]$

$570 = 8n + 2n^2$

$0 = n^2 + 4n - 285$

$0 = (n+19)(n-15)$

$\therefore n = 15\ ; n \neq -19$

n can never be negative therefore n is only equal to 15.

c) $16\,400 = \frac{5(3^n - 1)}{3-1}$

$32\,800 = 5(3^n - 1)$

$6\,560 = 3^n - 1$

$6\,561 = 3^n$

$\log_3 6\,561 = n$

$n = 8$

3.

a) $\sum_{n=1}^{5} \left(\frac{1}{2}\right)(4)^{n-1}$

$\therefore S_5 = \frac{\frac{1}{2}(4^5 - 1)}{4-1}$

$\therefore S_5 = 170\frac{1}{2}$

b) $\sum_{n=3}^{8} \frac{5n}{2}$

$\therefore S_6 = \frac{6}{2}\left[2\left(7\frac{1}{2}\right) + (6-1)\left(2\frac{1}{2}\right)\right]$

$\therefore S_6 = 3\left[27\frac{1}{2}\right]$

$\therefore S_6 = 82\frac{1}{2}$

c) $\sum_{n=6}^{18} 3 \cdot 2n$

$\therefore S_{13} = \frac{13}{2}[2(36) + (13-1)(6)]$

$\therefore S_{13} = \frac{13}{2}[144]$

$\therefore S_{13} = 936$

d) $\sum_{n=2}^{6} (4)\left(\frac{1}{2}\right)^{n-1}$

$\therefore S_5 = \frac{2\left(\left(\frac{1}{2}\right)^5 - 1\right)}{\frac{1}{2} - 1}$

$\therefore S_5 = 3\frac{7}{8}$

4.

a) $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore 182 = \frac{n}{2}[2(8) + (n-1)(6)]$

$\therefore 364 = n[16 + 6n - 6]$

$\therefore 0 = 16n + 6n^2 - 6n - 364$

$\therefore 0 = 6n^2 + 10n - 364$

$\therefore 0 = 3n^2 + 5n - 182$

$\therefore 0 = (3n + 26)(n - 7)$

$\therefore n \neq -\frac{26}{3}$ or $n = 7$ as it needs to be positive.

n also needs to be a whole number i.e. not a fraction

b) $S_n = \frac{a(r^n - 1)}{r-1}$

$\therefore 3\,069 = \frac{3(2^n - 1)}{2-1}$

$\therefore 3\,069 = 3(2^n - 1)$

$\therefore 1\,023 = 2^n - 1$

$\therefore 1024 = 2^n$

$\therefore \log_2 1024 = n$

$\therefore n = 10$

5.

a) $\sum_{n=1}^{100}(3n-5)$ Arithmetic series with:
T_1 = -2, T_2 = 1, T_3 = 4, therefore a = -2 and d =3

$S_n = \frac{n}{2}[2a+(n-1)d]$
$\therefore\ S_{100} = \frac{100}{2}[2(-2)+(100-1)(3)]$
$\therefore\ S_{100} = 50[293]$
$\therefore\ S_{100} = 14\ 650$

b) $\sum_{n=1}^{150}\left(\frac{1}{2}\right)(3)^{n-1}$ Geometric series with:
$T_1 = \frac{1}{2}; T_2 = \frac{3}{2};\ T_3 = \frac{9}{2},\ \ therefore\ a = \frac{1}{2}\ and\ r = 3$

$S_n = \frac{a(r^n-1)}{r-1}$
$\therefore\ S_{150} = \frac{\frac{1}{2}(3^{150}-1)}{3-1}$
$\therefore\ S_{150} = 9.25\ \times 10^{70}$

6.

a) a = -1, r = 3, therefore this is a geometric sequence.

$\sum_{n=1}^{k}(-1)(3)^{n-1}$

b) a = 2, r = , therefore this is a geometric sequence.

$\sum_{n=1}^{k}(2)\left(\frac{1}{2}\right)^{n-1}$

Activity 2.8

1.

a) $a = 2,\quad r = \frac{1}{3}$
$S_\infty = \frac{a}{1-r}$
$S_\infty = \frac{2}{1-\frac{1}{3}}$
$S_\infty = 3$

b) $a = 5.25,\quad r = \frac{1}{100}$
$S_\infty = \frac{a}{1-r}$
$S_\infty = \frac{5.25}{1-\frac{1}{100}}$
$S_\infty = 5\frac{10}{33}$

c) $a = 1, \quad r = \frac{1}{3}$

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{1}{1-\frac{1}{3}}$

$S_\infty = 1\frac{1}{2}$

d) $a = 1, \quad r = -\frac{1}{2}$

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{1}{1-\left(-\frac{1}{2}\right)}$

$S_\infty = \frac{2}{3}$

2. $T_1 = \frac{2}{3} \qquad T_2 = \frac{4}{3} \qquad T_3 = \frac{8}{3}$

$\therefore a = \frac{2}{3}$ and $r = 2$

$\therefore$ There is no sum to infinity because r > 1.

3. $S_\infty = \frac{a}{1-r}$

$\therefore 15 = \frac{3}{1-r}$

$\therefore 1 - r = \frac{3}{15}$

$\therefore r = \frac{4}{5}$

4. $S_\infty = \frac{a}{1-r}$

$\therefore \frac{7}{16} = \frac{a}{1-\frac{1}{8}}$

$\therefore a = \frac{7}{16} \times \frac{7}{8}$

$\therefore a = \frac{49}{128}$

5. $a = 2, \quad r = \frac{3}{5}$

$S_\infty = \frac{a}{1-r}$

$\therefore S_\infty = \frac{2}{1-\frac{3}{5}}$

$\therefore S_\infty = 5$

Therefore he can climb 5km before the incline is too steep and he will have to use climbing equipment.

Activity 2.9

1.

a) $a = 2, \quad r = 3$

$T_5 = (2)(3)^4$

$T_5 = 162$

b) $S_n = \frac{a(r^n-1)}{r-1}$

$2\,186 = \frac{2(3^n-1)}{3-1}$

$2\,186 = 3^n - 1$

$2\,187 = 3^n$

$\log_3 2187 = n$

$n = 7$

c) $S_3 = \frac{2(3^3-1)}{3-1}$

$S_3 = 26$ learners are affected after 3 days

2.

a) $T_3 = (5000)\left(\frac{11}{10}\right)^2$

$T_3 = 6050$

c) $T_{10} = 5000\left(\frac{11}{10}\right)^{10-1}$

$T_{10} = 11\,789.7$

$\therefore T_{10} = 11\,790$

You can't get 0.7 of a person so you will have to round off - remember to round up and not down.

b) $T_n = 15\,000 = (5000)\left(\frac{11}{10}\right)^{n-1}$

$\therefore 3 = \left(\frac{11}{10}\right)^{n-1}$

$\log_{\frac{11}{10}} 3 = n - 1$

$n - 1 = 11.5$

$n = 12.5$

Check: $T_{12} = 5000\left(\frac{11}{10}\right)^{11}$

$= 14\,265.58$

$T_{13} = 5000\left(\frac{11}{10}\right)^{12}$

$= 15\,692.14$

Therefore the population will reach 15 000 in the 12th year.